A new model for calculating permeability of natural fractures in dual-porosity reservoir

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Abstract: During the development of naturally fractured carbonate reservoirs, understanding the change in fracture permeability is the basis for production evaluation and scientific development. The conventional method of analyzing fracture permeability is to take core samples for laboratory experiments. This paper presents a new method to calculate the fracture permeability decrease using actual reservoir pressure data. The mathematical model of fracture permeability change with pressure is established based on material balance in the production process of a fractured reservoir. The model considers crossflow coefficient as well as compression coefficient. According to the results of the model, the fracture permeability decreases with decrease of the formation pressure, but the degree of decline is related to the crossflow coefficient and the compression coefficient. By using this model, the change in fracture permeability can be calculated under different production pressures. This provides a new method for stress sensitivity determination of fractured reservoirs.

Keywords: Natural fracture, carbonate reservoir, stress sensitivity, fracture permeability, fracture width, material balance.


1. Introduction

In recent years, carbonate reservoirs play an important role in oil and gas production. Carbonate reservoirs account for about 40% of the total global oil and gas reserves, and its production account for about 60% of the total production of oil and gas (Roeh et al., 1985; Halbouty, 2003). The stress sensitivity of carbonate reservoirs with natural fractures cannot be ignored. Many studies have shown that when the porosity and permeability of a rock change with pressure, the porosity stress sensitivity is negligible compared to the permeability stress sensitivity. Therefore, the present studies are mainly focused on the permeability stress sensitivity (Raghavan et al., 2002; Archer, 2008; Qiao et al., 2012).

Currently, researches on stress sensitivity are mainly focused on laboratory experiments. Stress sensitivity was first discovered in sandstone flow experiments by American scholars some researchers Fatt and Davis (1952). Then, some researchers (Jones, 1975; Jones et al., 1980; Walsh, 1981; Randolph et al., 1984; Jelmert et al., 1998) established a mathematical relationship between core permeability and effective stress, and found that the formula is applicable to naturally fractured carbonated reservoir and low permeability sandstone reservoir. Davies et al. (2001) compared the stress sensitivity characteristics of different permeability cores, and found that for unconsolidated cores with high permeability, the larger the porosity and permeability, the stronger the stress sensitivity. However, for some finely cemented cores with low permeability, the smaller the permeability, the stronger the stress sensitivity. Lei et al. (2007) established the quadratic polynomial between formation effective stress and permeability. The exponential relation between the permeability and effective stress of fractured reservoir was deduced and verified by Mckee et al. (1988). Thomas and Ward (1972) have showed that permeability decreases with increase in confining pressure but porosity changed a little. Buchsteiner et al. (1993) pointed out that the main reason for the decrease in permeability with an increase in effective stress of a fractured reservoir is caused by decrease in formation pressure. He also found that
the pore structure deformed under stress. When the effective stress increases, the fractures will be compressed and when the effective stress decreases, the fracture will be restored to a certain extent. Fractures are well developed in carbonate reservoirs, and they are mostly used as an important channel for oil and gas migration. The existence of fractures increases the permeability anisotropy. In the production process, change of pressure will lead to change of permeability (Palmer et al., 1996; Cho et al., 1999; Lorenz, 1999; Abass et al., 2007). There is no clear standard method used in analyzing the sensitivity of fractures. According to laboratory investigations, under normal pressure, the stress sensitivity of natural fractures is strong with great potential of aperture variation (Khan et al., 2000; Muralidharan et al., 2004; Petunin et al., 2011). Xiang et al. (2003) has studied the stress sensitivity of fracture-pore reservoir system, and believed that the stress sensitivity of artificial fractures, natural fractures and pore types of carbonate rock samples weakened sequentially. Shu et al. (2009) mainly discussed the variation laws of permeability, fracture width and effective stress. Tian (2014) has carried out a lab experiment and found that when permeability of fractured cores was measured in the process of increasing confining pressure with a constant pore pressure, the fracture is strongly sensitive to confining pressure change in laboratory, but only weakly sensitive to the reservoir pressure change in production. Duan et al. (1998) used elastic/plastic micro-contacting theory to study and analyze the topologic properties of both natural and many artificial fractures without any grinding, and a method of analyzing and evaluating stress sensitivity of dual-porosity reservoir with natural fractures is made on the basis of the investigation.

Scholars have developed some empirical formulas of stress sensitivity through laboratory experiments. These relationships mainly consist of the following 3 types: the power function, the multiplied power function and the exponential function (Li, 2006; Li, 2007; Guo et al., 2010). These relationships aided in the understanding of stress sensitivity. However, because the nature of oil and gas reservoir itself is stress sensitive, the studies conducted on stress sensitivity is still facing some problems: Firstly, the calculation of effective stress in the stress sensitivity experiments is still not unified. Secondly, studies on the stress sensitivity of fractured carbonate rocks are only based on experimental conclusions, and the analysis of fractures width is confined to static state.

In order to describe the variation of fracture permeability under different production pressure difference, crossflow and comprehensive compressibility are considered in this study. Based on material balance method, a new mathematical model of fracture permeability and production pressure is established by using dynamic production data. The model can be used to describe the variation of fracture width and permeability under different pressure difference, which plays an important role in the prediction of reservoir productivity.

2. Establishment of natural fracture permeability model based on the concept of material balance

Fractured reservoirs are mainly composed of fractures and matrix, therefore this paper uses the widely used physical model of Warren-Root. The model consists of fracture and matrix system. The matrix is the main reservoir space and the fracture is the main flow channel. During the development of the reservoir, oil flows from the matrix system to the fracture system, and then flows from the fracture system to the wellbore.

Fracture is the research objective of this paper. Before exploitation, the total volume of fluid in the fracture is \( lw_i B_i h \). During the production stage of the reservoir, the fluid in the fracture is continuously produced and the formation pressure decreases gradually. The amount of fluid produced by elastic energy is \( lw_i c_i (P_i - P_t) h \), and the remaining fluid is \( lw_i B_i h \). A pressure difference will occur when the formation pressure drops, which leads to the flow from the matrix to the fracture. The crossflow quantity is \( q_{cross} \). According to fluid volume balance, it is known that the sum of production from fracture by elastic energy and the residual fluid in the fracture should be equal to the sum of the total fluid flow in fracture and the amount of the crossflow. Thus the following equation was established:

\[
lw_i c_i (P_i - P_t) h = lw_i B_i h - lw_i B_i h + q_{cross}
\]

(1)

where \( l \) is total fracture length, \( cm; h \) is fracture height, \( cm; w_i \) is initial fracture width, \( cm; P_i \) is original formation pressure, \( MPa; B_i \) is volume coefficient under original formation pressure; \( P_t \) is the formation pressure after \( t \) hours production, \( MPa; w_t \) is the fracture width after \( t \) hours production, \( cm; B_t \) is volume coefficient after \( t \) hours production; \( q_{cross} \) is the crossflow quantity, \( cm^3 \).

Based on the general assumptions of flow modeling, the matrix medium is considered homogeneous and isotropic; and the fractures are uniformly distributed. For the crossflow phenomenon of the fracture system, the following equation was established:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{k_f}{\mu} \frac{\partial p_f}{\partial r} \right) \right] + q_{cross} = (\phi c_t) f \frac{\partial p_f}{\partial t}
\]

For the matrix system \((m)\), the \((\phi c_t)_m\) is used to represent the total elastic energy of the bulk matrix system. Based on the mass conservation principle and Darcy law, the relationship between \( q_{cross} \) and production pressure difference can be obtained:

\[
q_{cross} = - (\phi c_t) m \frac{\partial P_m}{\partial t} = \frac{k_m A_m}{\mu} \frac{P_t - P_i}{l} = \alpha(P_t - P_i)
\]

\[
\alpha = \frac{k_m A_m}{\mu l}
\]

(2)

where \( \alpha \) is the modifying factor of crossflow.

\[
c_t = \frac{1}{V_i} \frac{V_i - V_t}{P_t - P_i} = \frac{1}{B_t} \frac{B_t - B_i}{P_t - P_i} = \frac{1}{B_t} \frac{B_i - B_t}{P_t - P_i}
\]

(3)

So the oil volume factor after \( t \) hours production is:

\[
B_t = B_i (1 + c_t (P_i - P_t))
\]

(4)
The relationship between the fracture width and the production pressure difference is obtained by plugging equation (2) and (4) into equation (1).

\[ w_t = \frac{w_iB_i - \frac{w_i^3}{12}c_i(P_i - P_t) + \alpha(P_i - P_t)}{B_i(1 + c_i(P_i - P_t))} \]

\[ w_t = \frac{w_iB_i - w_i^3c_i(\Delta P) + \alpha(\Delta P)}{B_i(1 + c_i(\Delta P))} \]  

(5)

In this paper, the fractures in the Warren-Root model is simplified into a single fracture, the length of the fracture is \( l \), \( cm \); and the cross-sectional area is \( lw \), \( cm^2 \); Assuming that the flow direction is parallel to the fracture plane; then when the production pressure difference is \( \Delta P \). \( Mpa \); the flow equation is represented by the Poiseuille equation:

\[ Q = \frac{w^3l \Delta P}{12\mu L} \]  

(6)

The flow in the fractures is represented by the Darcy law:

\[ Q = \frac{k_fA\Delta P}{\mu L} \]  

(7)

From equations (6) and (7), the relationship between fracture permeability and fracture width can be obtained:

\[ k_f = \frac{w^3}{12A} = f_s \frac{w^3}{12} \]  

(8)

where \( k_f \) is fracture permeability, \( cm^2 \); \( f_s \) is fracture density, the total length of fractures in the unit percolation area, \( f_s = l/A \), \( cm/cm^2 \); \( l \) is fracture length on section, \( cm \); \( w \) is fracture width, \( cm \); \( L \) is core length, \( cm \); \( A \) is core cross sectional area, \( cm^2 \).

Before the production of oil well, the relationship between the initial fracture permeability and fracture width is:

\[ k_i = f_s \frac{w_i^3}{12} \]  

(9)

After \( t \) hours production, the relationship between the fracture permeability and fracture width is:

\[ k_t = f_s \frac{w_t^3}{12} \]  

(10)

So the ratio of \( k_t \) to \( k_i \) can be obtained as:

\[ \frac{k_t}{k_i} = (\frac{w_t}{w_i})^3 \]  

(11)

Considering the stress sensitivity of compressibility factor, Li (2007) has studied the comprehensive compressibility coefficient of rocks and found an exponential relation with production pressure difference.

\[ c_t = ce^{-\alpha(P_i - P_t)} \]  

(12)

where \( c \) is variation coefficient of compressibility with pressure; \( c \) is initial comprehensive compressibility.

\[ k_t = k_i [\frac{w_iB_i - w_i^3c_i\Delta P + \alpha\Delta P}{w_iB_i(1 + c_i\Delta P)}]^3 \]  

(13)

### 3. Comparative analysis of the new and exponential models

Presently, the exponential model is the most widely used mathematical model of stress sensitivity. The stress sensitivity coefficient is used to characterize the degree of stress sensitivity. The exponential model is as follows:

\[ k_t = k_i e^{-\alpha(P_i - P)} \]  

(14)

where \( k_t \) is the permeability of formation pressure is \( P \), \( 10^{-3} \mu m^2 \); \( k_i \) is the permeability of formation pressure, \( 10^{-3} \mu m^2 \); \( \alpha \) is stress sensitivity coefficient; \( P_i \) is original formation pressure, \( MPa \).

The \( \alpha \) in the formula is usually fitted by experimental data. In this paper, we used an oil field in Kazakhstan as an example, which belongs to fracture-pore type reservoir. The permeability of the core sample R2005 – 01206 is measured with overburden pressure, and the pressure sensitivity coefficient is obtained by fitting the permeability-pressure curve. Put \( \alpha \) into equation (14) to calculate the permeability under different production pressure difference. The data of the reservoir and the permeability of the core under different overburden pressures are shown in Table 1 and Table 2.

#### Table 1. Calculation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial formation Pressure ( P_i ) /MPa</td>
<td>39.4</td>
</tr>
<tr>
<td>Fracture permeability ( K_i ) /10(^{-3}) ( \mu m^2 )</td>
<td>5.81</td>
</tr>
<tr>
<td>Initial fracture width ( w_i ) /( \mu m )</td>
<td>150</td>
</tr>
<tr>
<td>Initial comprehensive compressibility ( c )/MPa(^{-1} )</td>
<td>0.002</td>
</tr>
<tr>
<td>Initial oil volume factor ( B_i )</td>
<td>1.47</td>
</tr>
</tbody>
</table>

By fitting the permeability-overburden pressure curve, the \( R^2 \) is 0.9785 and the stress sensitivity coefficient is 0.005. Plugging 0.005 into the exponential model, the permeability under different production pressure difference is obtained, and the results are compared with the permeability calculated by the new model in this paper. The results are shown in Table 3.

Table 3 shows that the difference between the permeability calculated by the new model and the exponential model is not significant, and the range of the percentage difference between the new and exponential models is between 0%-0.75%. For the conventional exponential model to be applicable in calculating the permeability, triaxial stress experiments are needed after field coring. Then to obtain the stress sensitivity coefficient, the permeability-overburden pressure curve must be fitted. We can use the coefficient to calculate the permeability under different production pressure difference. The exponential model is tedious and time-consuming. Compared to the exponential model, the new model considers the crossflow and comprehensive compressibility, timely monitoring of different production pressure difference of the permeability, avoiding the defects.
Table 2. Permeability of core R2005-01206 under different overburden pressures.

<table>
<thead>
<tr>
<th>Number</th>
<th>Sample depth /m</th>
<th>Overburden pressure /MPa</th>
<th>Permeability $10^{-3} \mu m^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2878.48</td>
<td>6.97</td>
<td>5.94</td>
</tr>
<tr>
<td>2</td>
<td>2878.48</td>
<td>13.52</td>
<td>5.66</td>
</tr>
<tr>
<td>3</td>
<td>2878.48</td>
<td>20.99</td>
<td>5.42</td>
</tr>
<tr>
<td>4</td>
<td>2878.48</td>
<td>27.91</td>
<td>5.21</td>
</tr>
<tr>
<td>5</td>
<td>2878.48</td>
<td>33.69</td>
<td>5.03</td>
</tr>
<tr>
<td>6</td>
<td>2878.48</td>
<td>41.5</td>
<td>4.89</td>
</tr>
<tr>
<td>7</td>
<td>2878.48</td>
<td>48.31</td>
<td>4.78</td>
</tr>
<tr>
<td>8</td>
<td>2878.48</td>
<td>55.14</td>
<td>4.68</td>
</tr>
</tbody>
</table>

Table 3. Comparison of permeability between the exponential model and new model.

<table>
<thead>
<tr>
<th>$\Delta P$ /MPa</th>
<th>Permeability $10^{-3} \mu m^2$</th>
<th>Percent difference(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exponential model $k_1$</td>
<td>New model $k_2$</td>
</tr>
<tr>
<td>0</td>
<td>5.81</td>
<td>5.81</td>
</tr>
<tr>
<td>1</td>
<td>5.78</td>
<td>5.76</td>
</tr>
<tr>
<td>2</td>
<td>5.75</td>
<td>5.72</td>
</tr>
<tr>
<td>3</td>
<td>5.72</td>
<td>5.68</td>
</tr>
<tr>
<td>4</td>
<td>5.69</td>
<td>5.65</td>
</tr>
<tr>
<td>5</td>
<td>5.67</td>
<td>5.62</td>
</tr>
<tr>
<td>6</td>
<td>5.64</td>
<td>5.60</td>
</tr>
<tr>
<td>7</td>
<td>5.61</td>
<td>5.58</td>
</tr>
<tr>
<td>8</td>
<td>5.64</td>
<td>5.57</td>
</tr>
<tr>
<td>9</td>
<td>5.55</td>
<td>5.55</td>
</tr>
<tr>
<td>10</td>
<td>5.53</td>
<td>5.54</td>
</tr>
</tbody>
</table>

of experimental static evaluation of reservoir stress sensitivity. The new model can be well related to actual production, and provides a new method for the calculation of productivity.

4. Analysis of influencing factors on fracture permeability

The compression coefficient and the fluid exchange between the matrix and fracture were considered basis on the principle of material balance, the mathematical relationship between fracture width, fracture permeability and production pressure difference were established. By using the model, we can quickly calculate the dynamic changes of fracture width and permeability under different pressure difference.

4.1 Relationship between fracture permeability, fracture width and production pressure difference

Based on the basic data of an oil field of Kazakhstan, the fracture permeability and fracture width are calculated. The curves of fracture permeability-production pressure difference, and fracture width-production pressure difference were drawn.

Figure 1 shows that whether or not the crossflow is considered, the fracture permeability decreases with the increase of production pressure difference. When the production pressure difference is small, the fracture permeability decreases rapidly with the increase of production pressure difference rendering the reservoir perform strong stress sensitivity. With further increase of production pressure difference, the change of fracture permeability tends to be slow; the reservoir performs weak stress sensitivity. Where there is crossflow between matrix and fracture, the fracture permeability is larger because the fluid in the fracture first flows into the wellbore. With increase in pressure difference, the matrix permeability is relatively low,

![Fig. 1. Relationship curves between fracture width and pressure difference.](image)
and the pressure drops slowly. The fracture permeability is high, and the pressure drops rapidly. Therefore, the matrix and the fracture will gradually produce pressure difference, the fluid in matrix will constantly flow into the fractures, and the decrease of fracture permeability becomes slow.

The fracture width decreases gradually with the increase in production pressure difference. In the actual production process, the production pressure difference increases with continuous fluid produced, so the reservoir is strongly compacted and easily deformed. The contact area of the two fracture surfaces get larger, which makes the fracture tend to close. With further increase of production pressure difference, the contact area reaches the maximum. The fracture will not be closed any more, and the change of fracture permeability will also slow down.

4.2 The influence of different compression coefficient on fracture permeability

Li (2007) pointed out that the stress sensitivity of rock is closely related to the comprehensive compressibility, and the higher the comprehensive compressibility, the stronger the stress sensitivity. The fracture permeability under different compression coefficient was obtained, and the curves of fracture permeability-production pressure difference were drawn.

Figure 2 shows that different compression coefficients have different effects on fracture permeability. The greater the compression coefficient, the more rapid the decrease in fracture permeability with production pressure difference and the stronger the stress sensitivity. In the actual production process, with increase in production pressure difference, the rock is compacted by overburden pressure, the particles are closely arranged, and the pore volume decreases. At the same time, the volume of rock particles is expanded by fluid in the pores as pressure drops. These decrease the compressibility and permeability.

4.3 The influence of different crossflow modifying factor on fracture permeability

Figure 3 shows that different crossflow modifying factors have different effects on fracture permeability. With an increase in production pressure difference and the smaller the crossflow modifying factor, the less fluid flows from the matrix into fractures, the more rapid fracture permeability decreases and the stronger the stress sensitivity. The greater the crossflow modifying factor, the easier the fluid exchange between the fracture system and the matrix system, and the faster the fluid exchange rate. With the continuous exploitation of crude oil, the formation pressure gradually decreases, leading to compaction of fractures, hence permeability of fractures decreases significantly.

4.4 The influence of different crossflow modifying factor on fracture permeability

In the fractured reservoir, the fracture width determines the seepage capacity. From Eq. (9) to Eq. (11), the $k_f$ and $w^3$ shows a liner relationship. However, the $k_f$ and $w$ have a good positive correlation based on the calculation result of Eq. (13).
Figure 4 shows that the fracture permeability decreases with decrease in fracture width. The relationship between them can be described as follows: The reduction of fracture permeability is the macroscopic performance of fracture width reduction, and the decrease of fracture width is the essential reason for the decrease of fracture permeability.

5. Conclusions

(1) Based on the material balance principle, the mathematical relations of fracture width, fracture permeability and production pressure difference are established. According to the calculation of reservoir parameters, the fracture width and fracture permeability can be obtained at any time. The results are compared with a laboratory empirical exponential model. The new model can be better combined with the actual production data; describing the dynamic changes in fracture width and fracture permeability as well as aid in the calculation of oil production capacity at any time.

(2) For natural fractured carbonate reservoirs, when the production pressure difference is small, the fracture permeability decreases rapidly. Then, with increase in pressure difference, the fracture permeability decreases slowly. Therefore, in the actual production process, taking corresponding production measures can reduce the fracture permeability reduction at a low production pressure stage.

(3) Under the same production pressure difference, the greater the compression coefficient, the smaller the crossflow modifying factor, the smaller the fracture width and the greater the decrease in fracture permeability.

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